Correcting the beam optics: Orbit Response Matrix analysis for the FAIR storage rings

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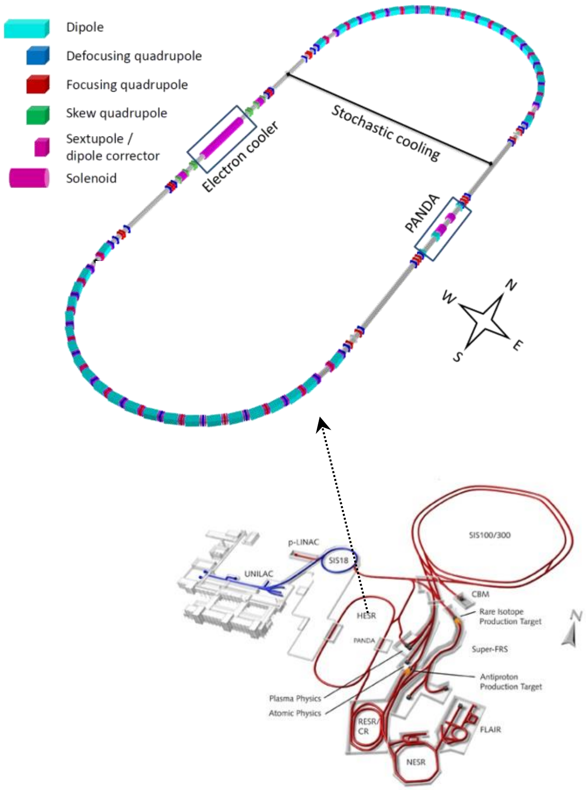
Introduction

- What is Orbit Response Matrix (ORM) analysis
- Why do we need it
- How was it implemented
- Application for FAIR storage rings.
- Results and observations

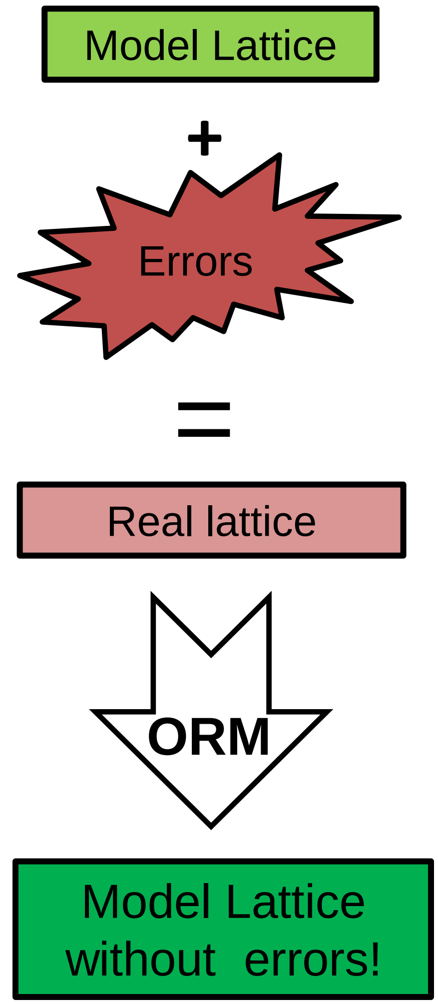


Why we need ORM analysis

- Real magnetic lattice has errors in comparison to the model:
  - Magnet rolls
  - Magnet misalignments
  - Wrong quadrupole settings
  - Calibration errors in dipole correctors and Beam Position Monitors

- When not corrected it causes:
  - Breaking the lattice periodicity
  - Increasing the harmful effects of the nonlinear resonances
  - Reducing the beam lifetime

- **ORM analysis**: search for the errors locations and values. **Restore the optics**.

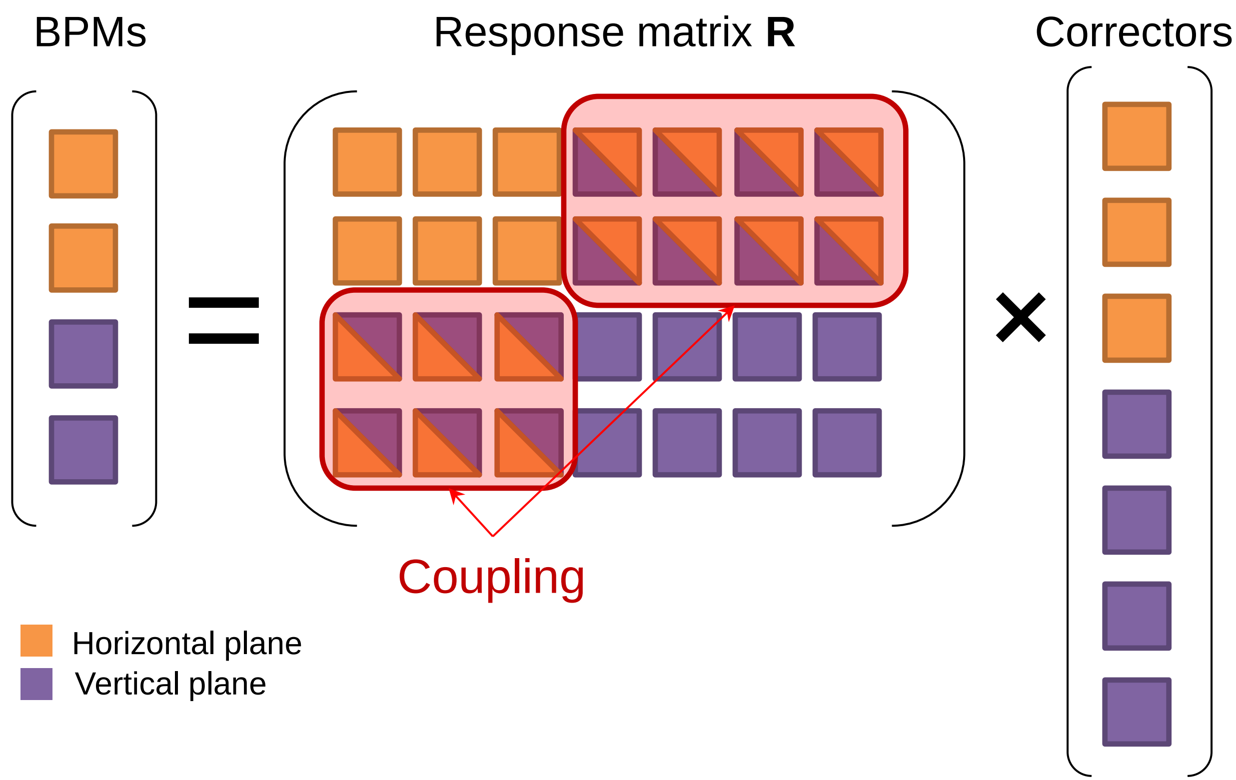


ORM analysis. Response

System	Knee	**Lattice**
Signal	Tiny hammer tap	**Dipole correctors kicks**
Reaction	Leg extension	**Beam offset at BPMs**
Internal data	Neuron pathway	**Machine parameters**

Orbit Response Matrix

$$\begin{pmatrix} x \\ y \end{pmatrix} = R\times\begin{pmatrix} \theta_x \\ \theta_y \end{pmatrix}$$



Minimize the difference between **observed** and **model** response matrices

Obtain the **errors** of the machine parameters

Observed (real) response matrix: $R_{ij}^{yy} = \frac{y_i}{\theta_{y,j}}$ (vertical plane)

Model (theoretical): $R_{ij} = \frac{\sqrt{\beta_i\beta_j}}{2\sin \pi Q}\sin\left(2\pi|\varphi_i - \varphi_j| - \pi Q\right)$

β – beta-function, φ – betatron phase, Q – betatron tune

Tools

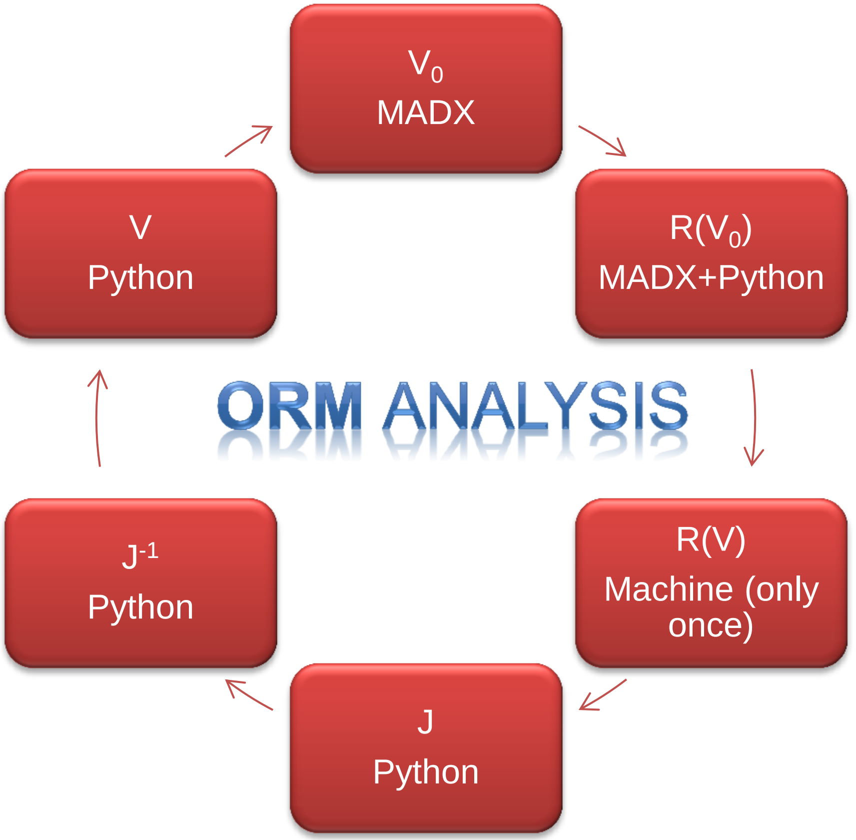
- CERN MAD-X accelerator code
  - Twiss parameters computation
  - Beam orbit computation
  - Nonlinear beam dynamics

- Python programming language + PyQt + linear algebra libraries
  - General algorithm
  - Testing
  - Graphical User Interface
  - Lattice parser
  - Plotting
  - Singular Value Decomposition

Iteration process

- $\boldsymbol{V}$ – quadrupole gradients vector.
- $\boldsymbol{R} = \boldsymbol{f}(\boldsymbol{V})$ – orbit response vector
- $\boldsymbol{R}(\boldsymbol{V}) \approx \boldsymbol{R}(\boldsymbol{V_0}) + \boldsymbol{R}'(\boldsymbol{V_0})(\boldsymbol{V} - \boldsymbol{V_0})$
- $\boldsymbol{R}'(\boldsymbol{V_0}) = \boldsymbol{J} = \begin{bmatrix} \frac{\partial R_1}{\partial V_1} & \cdots & \frac{\partial R_1}{\partial V_k} \\ \vdots & \ddots & \vdots \\ \frac{\partial R_{mn}}{\partial V_1} & \cdots & \frac{\partial R_{mn}}{\partial V_k} \end{bmatrix}$
- $\boldsymbol{V} = \boldsymbol{V_0} + \boldsymbol{J}^{-1} \times [\boldsymbol{R}(\boldsymbol{V}) - \boldsymbol{R}(\boldsymbol{V_0})]$

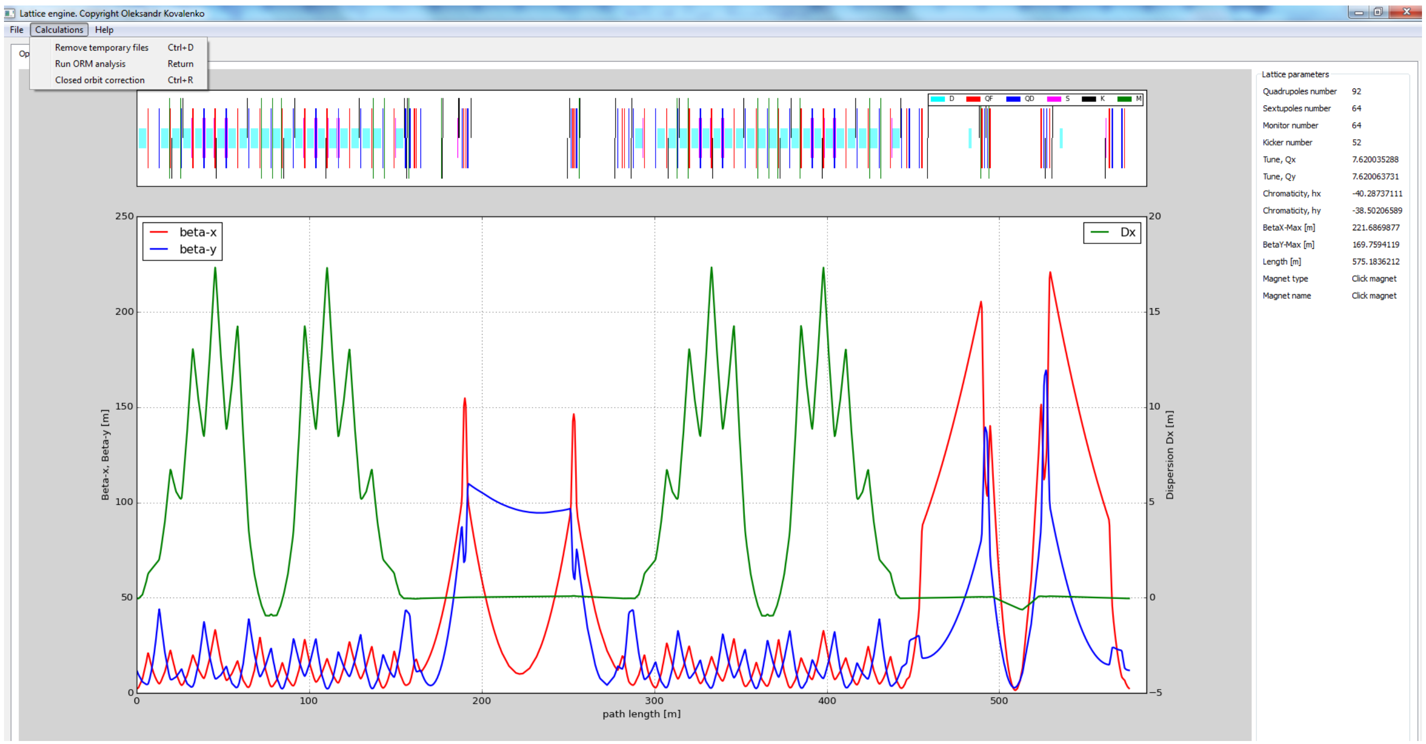
χ^2_{red} – estimation of fit quality



Cycle

1. $\boldsymbol{V_0}$ – first guess for the gradients is known from design
2. $\boldsymbol{R}(\boldsymbol{V_0})$ – model response vector computed with MADX
3. $\boldsymbol{R}(\boldsymbol{V})$ – observed response vector found by varying correctors
4. $\boldsymbol{J}$ – Jacobian computed by varying gradients
5. $\boldsymbol{J}^{-1}$ – pseudoinverse Jacobian calculated via SVD
6. $\boldsymbol{V}$ – new quadrupole gradients
7. $\boldsymbol{V_0} = \boldsymbol{V}$. Back to the first step

Program

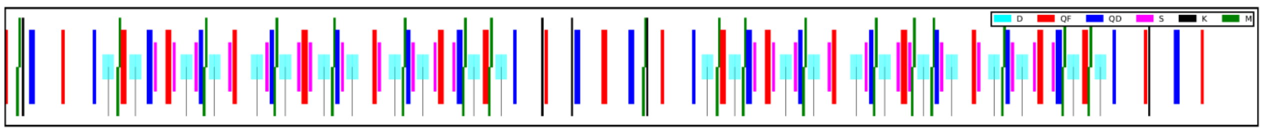


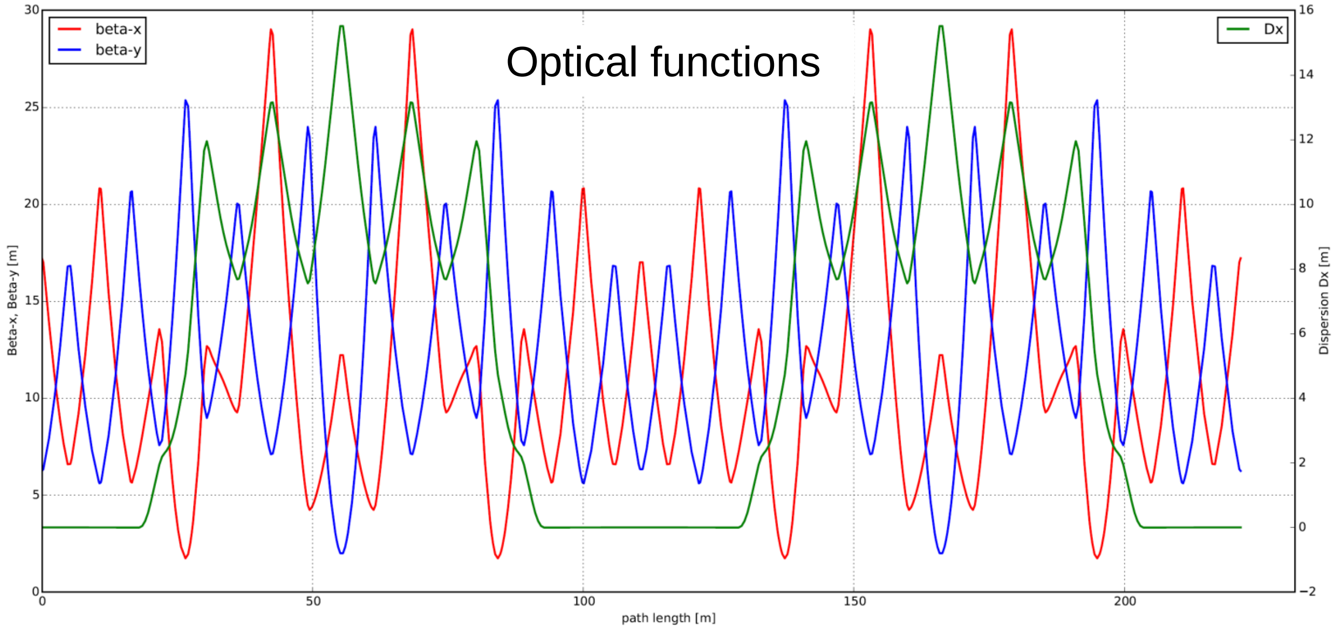
- Lattice statistics
- BPM readout errors on/off
- Coupling on/off
- Search for the nonlinear errors (e.g. in sextupole gradients).

Results

CR storage ring

Machine lattice





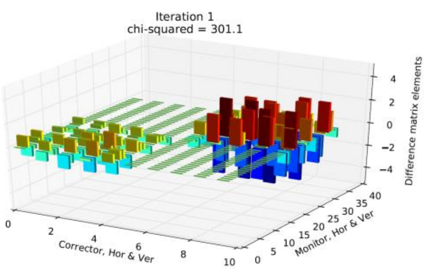
Lattice	
Length	221 m
BPMs	18
Correctors, hor/ver	29/21
Quadrupole families	11

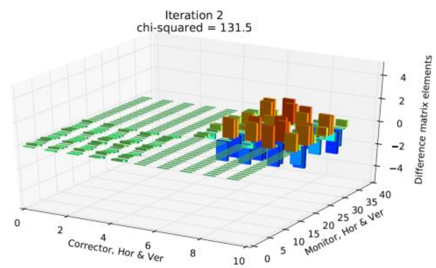
Orbit Response Matrix minimization in the CR

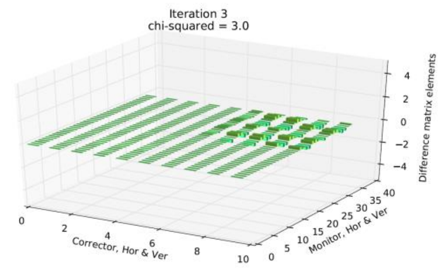
- 10 dipole correctors
- ±5% uniform quadrupole gradient errors
- Perfect BPMs

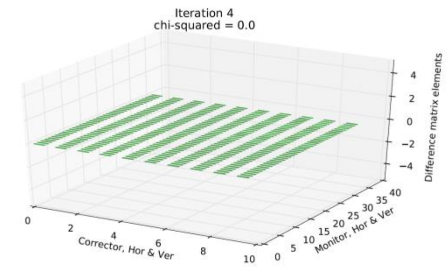
1 | 2 | 3 | 4

ORM discrepancy



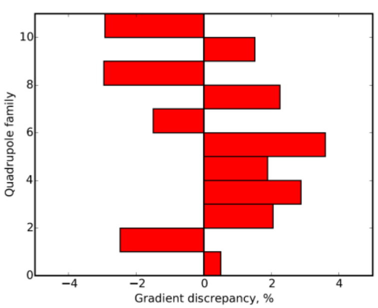


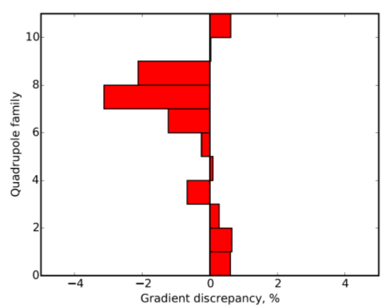


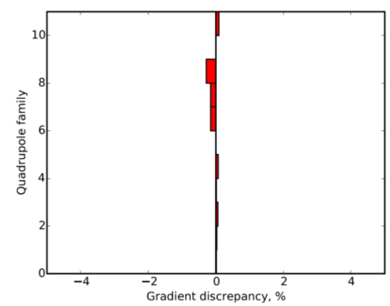


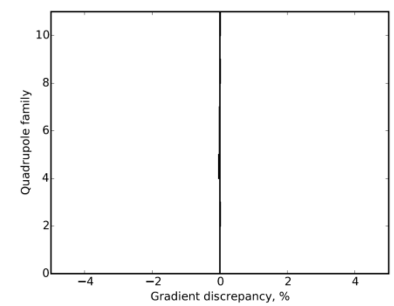
4 iterations. Errors found. Optics restored.

Gradient discrepancy

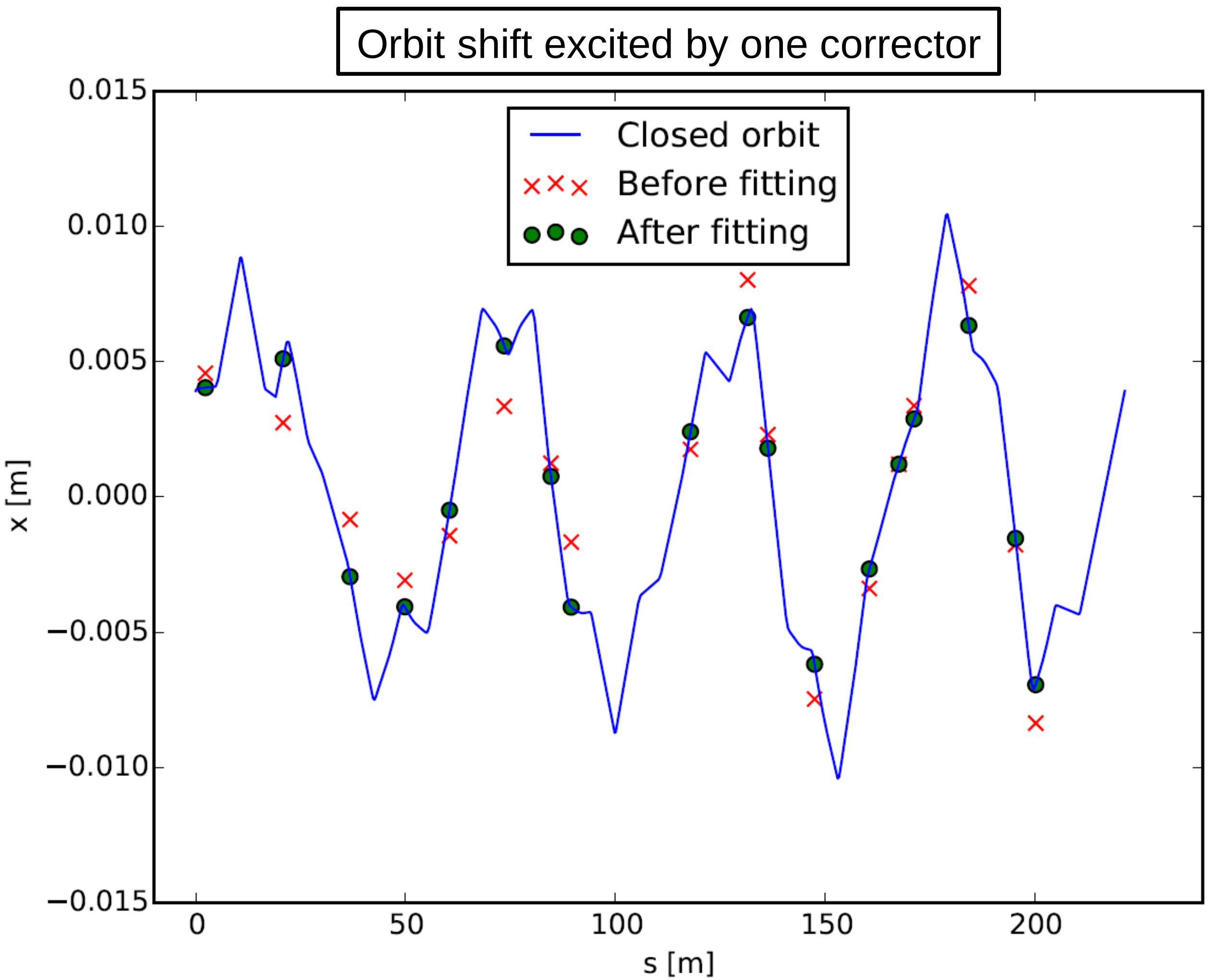






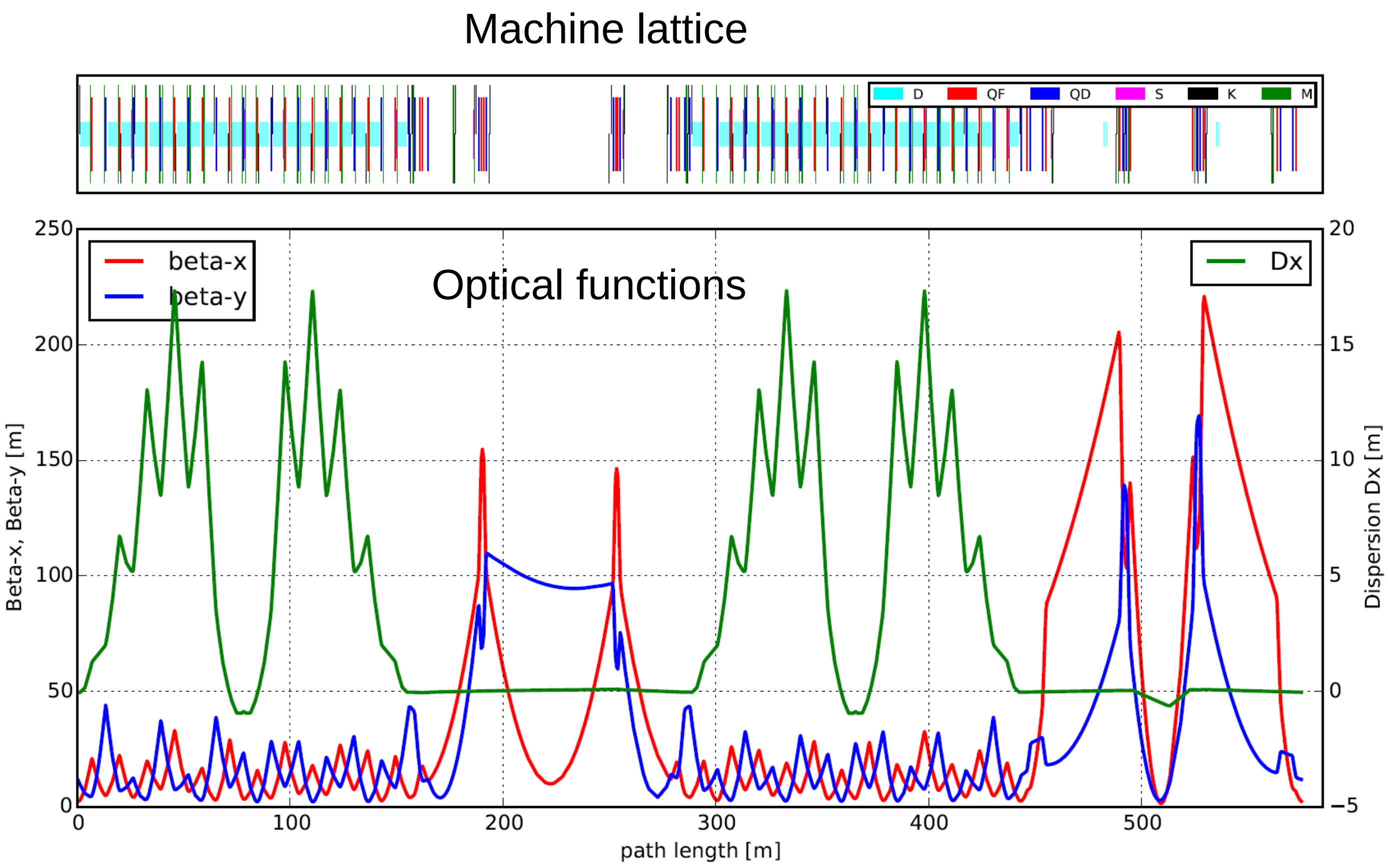


ORM analysis. Orbit fit. CR



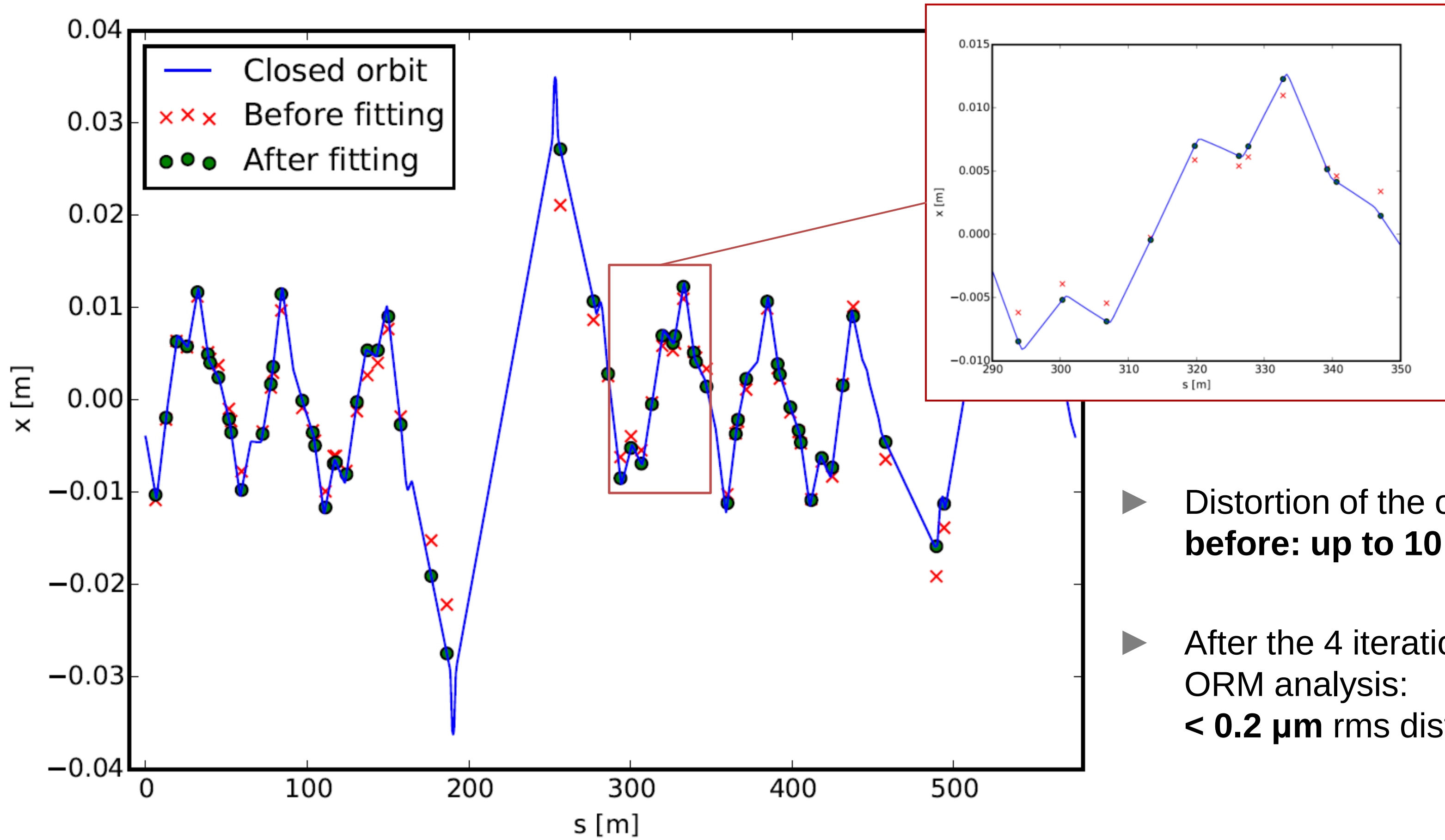
- 50 dipole correctors. Random quadrupole gradient errors in the range ±5%.
- The distortion of the orbit before the fit is up to **3 mm**.
- Three iterations, about one minute of computation time
- All the errors of the quadrupole gradients were found almost exactly!
- The rms difference between the model and the measured orbit (green cirlces) after the fit is **less than 1 μm**.

HESR storage ring



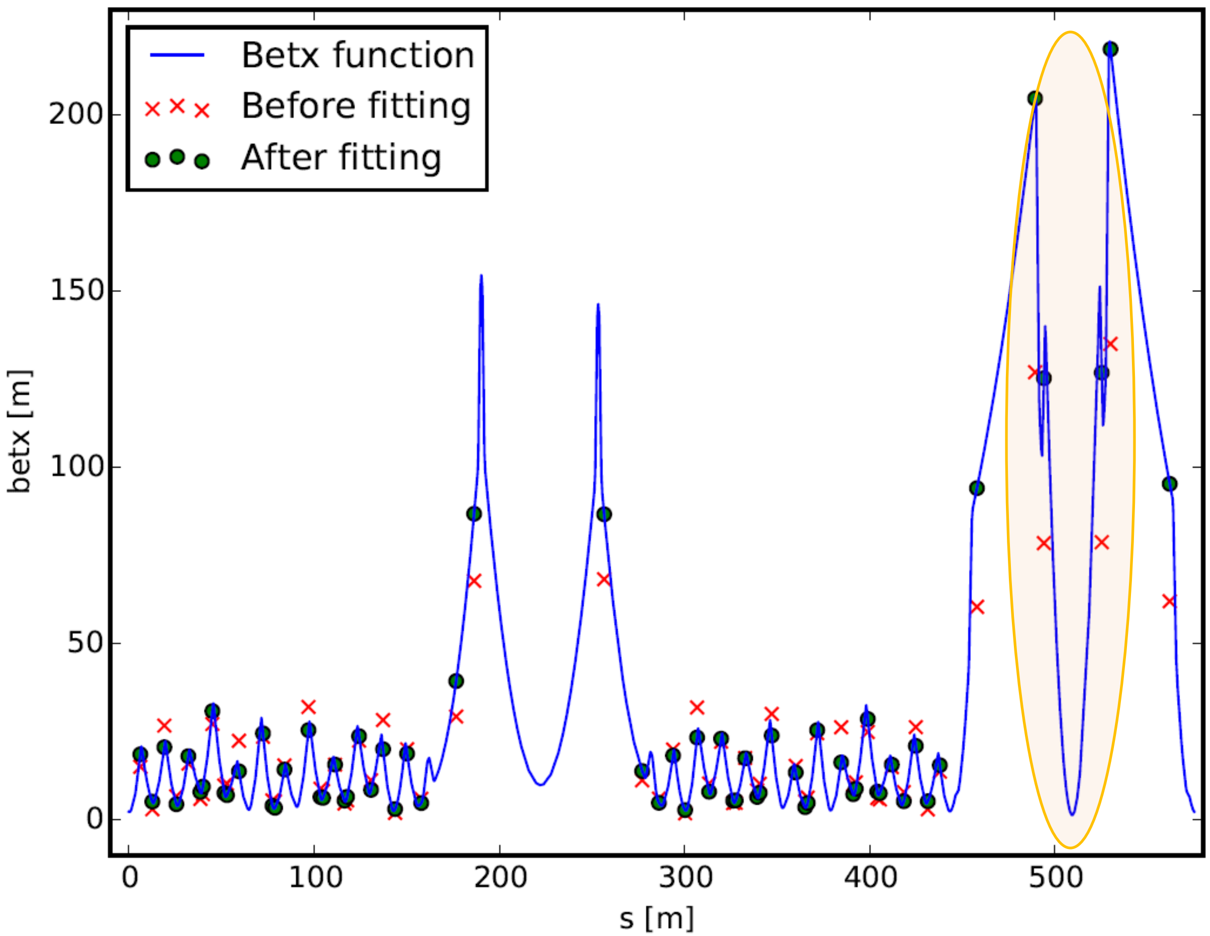
Lattice	
Length	575 m
BPMs	64
Correctors, hor/ver	26/26
Quadrupole families	12

Fitting the orbit. HESR



- Distortion of the orbit **before: up to 10 mm**
- After the 4 iterations of ORM analysis: **< 0.2 µm** rms distortion

Beta function fit. HESR



- Beta function: focusing, beam transverse size
- **Before** fitting **> 70%** (in the target section) discrepancy between model and real beta function
- **After** ORM fitting: **< 0.03% rms** deviation of beta-function

Observations

- **Sensitivity**. BPM readout errors.
  - uniformly distributed BPM readout error in the range ±1 mm . After 3-4 iterations the ORM code converged with $\chi^2 = 0.99$ for the CR and $\chi^2 = 0.97$ for the HESR. The quadrupole gradients were found with an rms error ≈0.5% (BPM noise level).
- **Singal-to-noise ratio**. Use the maximum possible corrector kicks.
  - CR: the orbit offsets at BPMs were limited by the maximum strength of the dipole correctors.
  - HESR: the limiting factor was the magnet aperture (full aperture in most of the magnets is only 89 mm)
  - This approach might however fail when there are strong nonlinearities in the BPMs leading to large readout errors for large orbits offsets (at the edges of BPMs).
- **Data for the response matrix**. Use any type of data you can obtain.
  - **Dispersion**: varying frequency in the RF accelerating cavity, measure beam offsets.
  - **Phase advances**
  - **Tunes** etc.

Conclusions

- Response matrix analysis is a powerful technique to track down the errors in the lattice.
- Can be used during beam commissioning as well as during routine machine operation
- Type of magnetic elements, the errors of which can be corrected:
  - normal and skew quadrupole gradients
  - calibration factors of BPMs and correctors
  - sextupole gradients
- Successful proof of concept for CR and HESR FAIR storage rings. Faulty optics was completely restored.

Next steps

- Applying the ORM analysis for SIS100
- Test of the ORM analysis for the existing machines
  - CRYRING (GSI), COSY (FZ Jülich), CSR (Lanzhou)
- ORM analysis into Java